# CLAIMS

1. An apparatus for determining field-dependent characteristics comprising:
   - A) a storage medium containing canonical quadratures; and
   - B) a computation circuit responsive to signals representing the shape of a boundary that includes geometrical singularities of different angles to:
     - i) divide the boundary into problem intervals;
     - ii) for each of a number of target nodes, perform a numerical integration over the boundary of an integrand defined thereon by, for at least some combinations of target node and problem interval that contains a geometrical singularity that induces a singularity in the integrand, performing the integration for that target point node over that problem interval in accordance with a canonical quadrature chosen from among the canonical quadratures independently of what, within a given angle range, the value of that geometric singularity's angle is;
     - iii) determine the field-dependent characteristic at least in part by employing the results of the numerical integration thus performed; and
     - iv) generate an output signal indicative of the characteristic thus determined.

2. An apparatus as defined in claim 1 wherein:
   - A) each of the stored quadratures is associated with a respective position of a target node or a target-node region with respect to a canonical integration interval and is based on the integration, over the canonical integration interval, of the product of a kernel function and a density function, to both of whose domains the canonical interval belongs;
   - B) each of a plurality of the quadratures is associated with a respective set of at least one density-singularity location on the canonical interval;

C) the value of the kernel function depends on the relative target-node position associated with that quadrature,

D) the density function is independent of the target node's position and exhibits a singularity only at each density-singularity position associated with that quadrature; and

E) the quadrature performs the integration for that target point node over a problem interval by mapping the problem interval to the canonical interval and selecting therefor a said canonical interval associated with a density-singularity position at each point on the canonical interval to which a geometric singularity on that problem interval is thereby mapped.

3. An apparatus as define in claim 1 wherein the computation circuitry:

A) applies a Fast Multipole Method (FMM) using far-field quadratures to provide an FMM result;

B) identifies one or more target points for which the contribution to the FMM result from one or more intervals does not achieve a desired accuracy;

C) removes from the FMM result for each such target point the contribution from each such interval based on the determined one or more points,

D) performs the canonical-quadrature-integration operation for such intervals to obtain a replacement contribution, and,

E) adds the second contribution to the FMM result.

4. An apparatus as defined in claim 1 wherein the number of angle ranges is no more than one thousand.

5. An apparatus as defined in claim 4 wherein the number of angle ranges is no more than one hundred.

6. An apparatus as defined in claim 5 wherein there is only a single angle range.